Available online at www.sciencedirect.com

ScienceDirect

Journal of Sound and Vibration 324 (2009) 954–961

JOURNAL OF SOUND AND VIBRATION

www.elsevier.com/locate/jsvi

Line spectra reduction and vibration isolation via modified projective synchronization for acoustic stealth of submarines

Guilin Wen*, Yuanzhi Lu, Zhiyong Zhang, Chuanshuai Ma, Hanfeng Yin, Zhong Cui

State Key Laboratory of Advanced Design and Manufacture for Vehicle Body, Institute of Space Technology, Hunan University, Changsha City 410082, China

Received 14 February 2008; received in revised form 7 January 2009; accepted 22 February 2009
Handling Editor: L.G. Tham
Available online 31 March 2009

Abstract

The potential of chaotifying vibration isolation systems to reduce line spectra of underwater vehicle and improve its capability of concealment have recently reported. Notice that for an isolation system, the amplitude of chaotifying vibration is, in general, larger than that of its specified periodic motion subject to the inherent characteristic of chaotic behavior. Therefore, there exists a conflict between the line spectra reduction based on chaotifying vibration responses and the capability of vibration isolation. In this paper, a method based on the modified projective synchronization (MPS) is developed to improve vibration isolation while the chaotifying vibration signals with broad band frequency are utilized to achieve line spectra reduction. The key to this solution is the scaling factors of MPS which enable us to proportionally diminish the vibration amplitudes of the isolated equipment. The feasibility of the methodology in practical engineering is illustrated with the application to acoustic stealth of submarines.

1. Introduction

The past decade has witnessed a surge in the potential application of chaos synchronization in secure communication, liquid mixing, biological systems (see Refs. [1–5] and the references therein). For example, with the application of chaos synchronization in secure communication [3–5], the information masked with chaotic signals can be recovered through a synchronization process. However, the positive value of chaos synchronization has not been convincingly accepted in mechanical engineering field. In mechanical engineers' view, the inherent features of chaos have been considered as certain source of destructivity for structure [6].

The seminal work [7–9] on the method of chaotic vibration isolation for reduction of line spectra in the water-born noise of marine vessels is very interesting. Line spectra reduction is of great significance for

*Corresponding author. Tel.: +86 731 8821482; fax: +86 731 8822051.
E-mail address: wenguilin@yahoo.com.cn (G. Wen).

0022-460X/$ - see front matter
doi:10.1016/j.jsv.2009.02.049

improving the acoustic stealth of submarines [7,9]. The periodic line spectrum contained in the radiated noise of submarines is mainly produced by periodic excitation of machines such as diesel engine. An onboard vibration isolation system working in a chaotic state can effectively reduce the intensity of line spectra at the primary harmonic frequencies due to the spectrum of chaotic vibration with broad band frequency. It should be mentioned that there exist two aspects of difficulties in the application of the method to practical engineering. The one is how to sustain the persistent chaotic motion of a vibration isolation system; the other is how to preserve the performance of vibration isolation as the system becomes chaotic. It is well known that according to the traditional mechanical design theory, it is difficult to design a vibration isolation system with enough large chaotic parameter range for the robustness of chaotic motion. Liu et al. [8] presented the feedback control for chaotifying vibration isolation systems based on the calculation of the Lyapunov exponent. Unfortunately, the classical chaos criteria such as calculation of the Lyapunov exponent and Melnikov method are infeasible in real time control of chaos under complex submarine circumstances. Yu et al. [9] proposed the chaos synchronization method to make the chaotic signal to drive a vibration isolation system. The intensities of line spectra at the primary harmonic frequencies are effectively reduced. However, although the improvement of vibration isolation at a particular case of parameters is displayed by numerical simulation, there is no guarantee in theory for the synchronization scheme to improve the isolation performance of vibration while the chaotifying motion is used to reduce line spectra. This problem gives the remarkable limitation of the method in practical engineering. After all, the vibration isolation quality is a matter of importance for isolated equipment in submarines. It is a well-known fact that there are numerous unstable periodic trajectories embedded in a chaotic attractor and the amplitude of a particular period solution is not necessarily less than the one of chaos attractor. This characteristic can be seen from the bifurcation plots of period-doubling bifurcation cascades [10] or torus-doubling bifurcations [11,12]. In theory, neither the traditional mechanical design theory nor the chaos synchronization in a general sense [9,13] is capable of solving the conflict of isolation performance in applications of chaos to reduce line spectra due to the inherent characteristic of chaotic behavior.

In this paper, we propose a control method for modified projective synchronization (MPS) [14,15] as a complete solution to realize the optimum objectives of both line spectra reduction and vibration isolation. Different from complete synchronization or identific synchronization [13] where two coupled chaotic systems exhibit identical oscillations, the scaling factors [16–18] of MPS is capable of proportionally reducing the driven vibration amplitude and improving isolation performance as our desired design objective. Many techniques for handling chaos synchronization such as nonlinear observer control [18], OGY method [19], time-delay feedback approach [20], PID control [21] and adaptive control [22] have been reported. Notice that the control scheme proposed in this paper is easily generalized to the vibration systems with uncertain parameters by utilizing the adaptive control in Refs. [14,23], or the function MPS with time-varying scaling factors [24]. The objective of this paper is to propose the methodology on the application of MPS to vibration isolation and address the problem about the positive value of chaos synchronization in mechanical engineering field. In addition, some implementation problems of the proposed methodology in practical engineering are discussed in brief.

2. MPS of vibration isolation system

A vibration isolation system in submarines consists of the isolated equipment and the base. The model of the type of vibration isolation systems can be considered as a two-degree-of-freedom mass–spring system [9]. The control scheme based on MPS for line spectra reduction in the water-born noise and vibration isolation is illustrated in Fig. 1.

In this control scheme, the vibration isolation system with adjustable damper and spring stands for the response system and the Duffing system is selected as the driving chaotic system. In practical engineering, the chaotic driving signals of Duffing system can be generated by a simple analog circuit instead of a cumbersome mechanical driving system.

The closed-loop system for synchronization can be formulated as follows:

$$\dot{x}_1 = x_2, \tag{1}$$

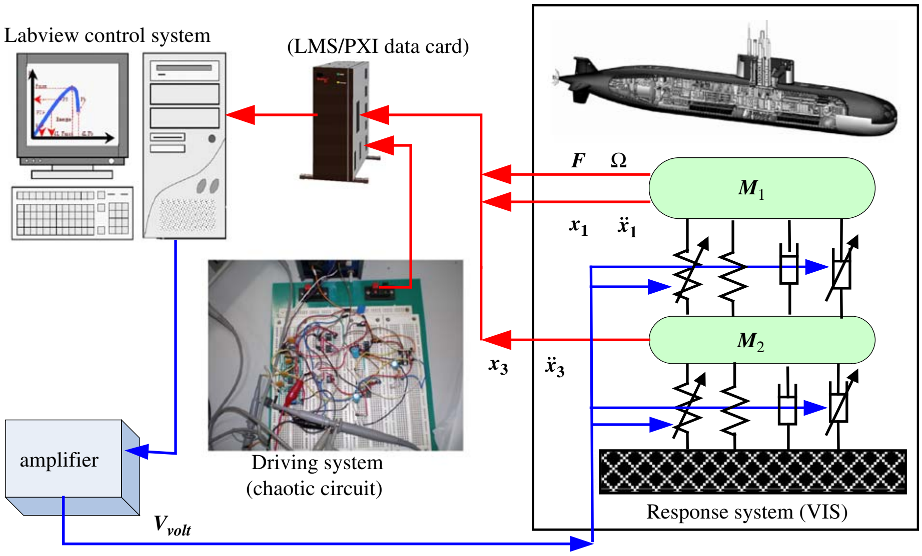

Fig. 1. Control scheme for line spectra reduction and vibration isolation.

$$\dot{x}_2 = f_2(x_1, x_2, x_3, x_4) + u_1, \tag{2}$$

$$\dot{x}_3 = x_4, \tag{3}$$

$$\dot{x}_4 = f_4(x_1, x_2, x_3, x_4) + u_2, \tag{4}$$

$$\dot{x}_5 = x_6, \tag{5}$$

$$\dot{x}_6 = f_6(x_5, x_6), \tag{6}$$

where f_i, $i = 2, 4, 6$, are nonlinear functions. The motions of the original isolation system are described by the following Eqs. (1)–(2) and (3)–(4) with $u_1 = 0$ and $u_2 = 0$, respectively. The Duffing system as the driving chaotic system is described by Eqs. (5)–(6). The control inputs u_1 and u_2 are selected as the sense of forces in order to meet with the requirement of control implementation in mechanical engineering. The feedback control scheme is expressed by,

$$u_1 = -f_2 + \alpha_1 f_6 - \lambda_1 e_1 - \lambda_2 e_2, \quad u_2 = -f_4 + \alpha_2 f_6 - \lambda_1 e_3 - \lambda_2 e_4, \tag{7}$$

where the α_i, $i = 1, 2$, denote the scaling factors for MPS [14–18]. The gains $\lambda_i > 1$, $i = 1, 2$, are positive real constants and are available for speeding up the convergence of the synchronization. Notice that our attempt in this paper is to show the positive value of chaos synchronization in mechanical engineering field, not to develop new control methods. With loss of generality, we set $\lambda_2 = \lambda_1 + 1$ in Eq. (7) for simplifying the design of control. The synchronization error system is expressed by

$$\dot{e} = G(e), \tag{8}$$

where $e = (e_1, e_2, e_3, e_4)^{\mathrm{T}}$, and the synchronization errors $e_1 = x_1 - \alpha_1 x_5$, $e_2 = x_2 - \alpha_1 x_6$, $e_3 = x_3 - \alpha_2 x_5$ and $e_4 = x_4 - \alpha_2 x_6$. It is followed from Eqs. (1), (3) and (5) that $\dot{e}_1 = e_2$ and $\dot{e}_3 = e_4$.

For system (8), consider the following Lyapunov function:

$$V(t) = (e_1 + e_2)^2/2 + \lambda_1 e_1^2 + (e_3 + e_4)^2/2 + \lambda_1 e_3^2, \tag{9}$$

By calculating the derivative of $V(t)$ along with the system (8) and utilizing $\dot{e}_1 = e_2$, $\dot{e}_3 = e_4$, and $\lambda_2 = \lambda_1 + 1$, we can obtain,

$$
\begin{aligned}
\dot{V}(t) &= (e_1 + e_2)(\dot{e}_1 + \dot{e}_2) + 2\lambda_1 e_1 \dot{e}_1 + (e_3 + e_4)(\dot{e}_3 + \dot{e}_4) + 2\lambda_1 e_3 \dot{e}_3 \\
&= (1 + 2\lambda_1) e_1 \dot{e}_1 + e_1 \dot{e}_2 + e_2 \dot{e}_1 + e_2 \dot{e}_2 + (1 + 2\lambda_1) e_3 \dot{e}_3 + e_3 \dot{e}_4 + e_4 \dot{e}_3 + e_4 \dot{e}_4 \\
&= (1 + 2\lambda_1) e_1 \dot{e}_1 + e_1(\dot{x}_2 - \alpha_1 \dot{x}_6) + e_2 \dot{e}_1 + e_2(\dot{x}_2 - \alpha_1 \dot{x}_6) \\
&\quad + (1 + 2\lambda_1) e_3 \dot{e}_3 + e_3(\dot{x}_4 - \alpha_2 \dot{x}_6) + e_4 \dot{e}_3 + e_4(\dot{x}_4 - \alpha_2 \dot{x}_6) \\
&= (1 + 2\lambda_1) e_1 \dot{e}_1 + e_1 e_1(f_2 + u_1 - \alpha_1 f_6) + e_2 \dot{e}_1 + e_2(f_2 + u_1 - \alpha_1 f_6) \\
&\quad + (1 + 2\lambda_1) e_3 \dot{e}_3 + e_3(f_4 + u_4 - \alpha_2 f_6) + e_4 \dot{e}_3 + e_4(f_4 + u_4 - \alpha_2 f_6) \\
&= -\lambda_1 e_1^2 - \lambda_1 e_2^2 - \lambda_1 e_3^2 - \lambda_1 e_4^2.
\end{aligned} \tag{10}
$$

It follows from Eqs. (9) and (10) that the equilibrium solution $e = 0$ of the synchronization error system (8) is Lyapunov stable [25], i.e., $e_i \to 0$, $i = 1, 2, 3, 4$, as $t \to \infty$. In other words, the motions of the isolated equipment and the base are proportionally synchronized as the motion of Duffing chaotic system according to the scaling factors α_1 and α_2. Similar to the detailed analysis presented in Refs. [7–9], one can show that the chaotification based on the projective synchronization is used to reduce line spectra of the isolation system. It follows from Eq. (8) that with proper choice of the scaling factors α_1 and α_2, MPS can make the vibration amplitudes of the isolated equipment synchronize to the proportionally diminished or enlarged driving signals. The scaling factors α_1 and α_2 are capable of improvement of isolation performance by suppressing the vibration of the isolated equipment as degree as we desire in engineering. It should be mentioned that besides the signal-diminished function of α_1 and α_2, their signal-enlarged function sometimes is helpful for the synchronization. For example, if the amplitudes of the chosen driving system are too small to match with the requirement of a mechanical system with persistent chaotic motion, the scaling factors α_1 and α_2 may properly enlarge the driving signals.

3. Numerical example

To demonstrate the effectiveness of line spectra reduction and improvement of vibration isolation, we take the following functions f_j, $j = 2, 4, 6$, in the close-loop system (1)–(6) as an example,

$$f_2 = -0.2(x_2 - x_4) - (x_1 - x_3) + (x_1 - x_3)^2 - (x_1 - x_3)^3 + 20\cos(3.9311t), \tag{11}$$

$$f_4 = 0.04(x_2 - x_4) - 0.04x_4 - 0.4x_3 + 0.2(x_1 - x_3) - 0.2(x_1 - x_3)^2 + 0.2(x_1 - x_3)^3, \tag{12}$$

$$f_6 = -x_5 - x_5^3 - 0.1x_6 + 9\cos(3.9311t) + 2. \tag{13}$$

For the sake of comparability, the system parameters are taken from Ref. [9]. For the synchronization control, we take $\alpha_1 = 1/10$, $\alpha_2 = 1/30$, $\lambda_1 = 30$ and $\lambda_2 = 31$ in Eq. (7) and already show that the equilibrium solution $e = 0$ of the synchronization error system (8) is Lyapunov stable. The numerical simulations of the close-loop system is implemented with the initial state: $x_1(0) = x_2(0) = 0.1$, $x_3(0) = x_4(0) = -0.1$ and $x_j(0) = 0$, $j = 5, 6$. The total time and step of the fourth-order Runge–Kutta integration method are 800 and 0.01, respectively. In Fig. 2, the time histories of the errors e_i, $i = 1, 2, 3, 4$, show the fact that the synchronization is achieved quickly.

The typical chaotic response of the isolated equipment under control is shown in Fig. 3(a) in comparison with the periodic response of the original isolation system illustrated in Fig. 3(b). The phase portraits of the isolated equipment are shown in Fig. (4). It is obvious that the vibration amplitudes in the controlled case are much less than those in the uncontrolled case. Notice that the method in Ref. [9] has no theory for determining the appropriate selection of the driving parameter to suppress vibration amplitudes. However, for the method herein, the scaling factors of MPS enable us to proportionally diminish the vibration amplitudes as degree as we desire. The inherent characteristic of MPS is the key of this perfect solution to achieve both the reduction of line spectra and the improvement of vibration isolation.

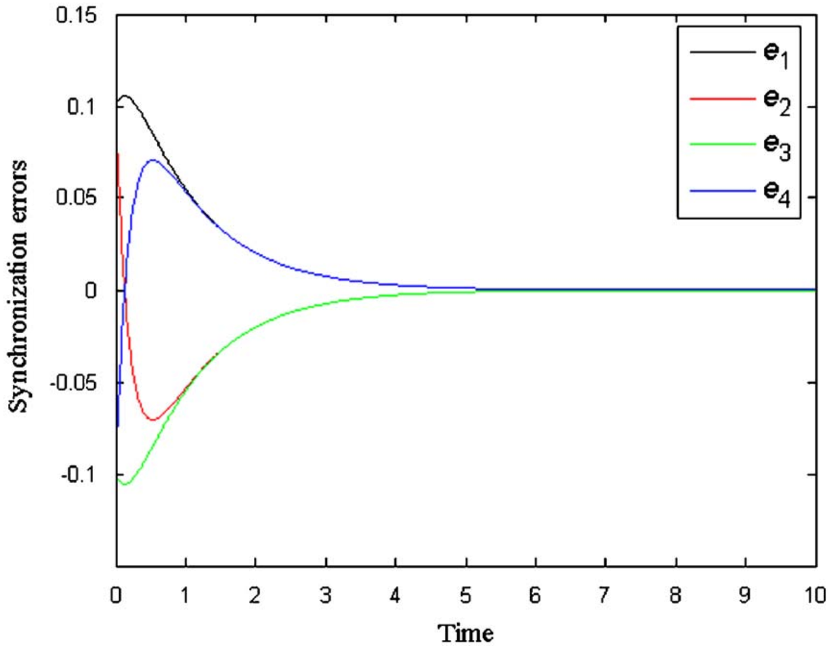

Fig. 2. The time histories of the errors.

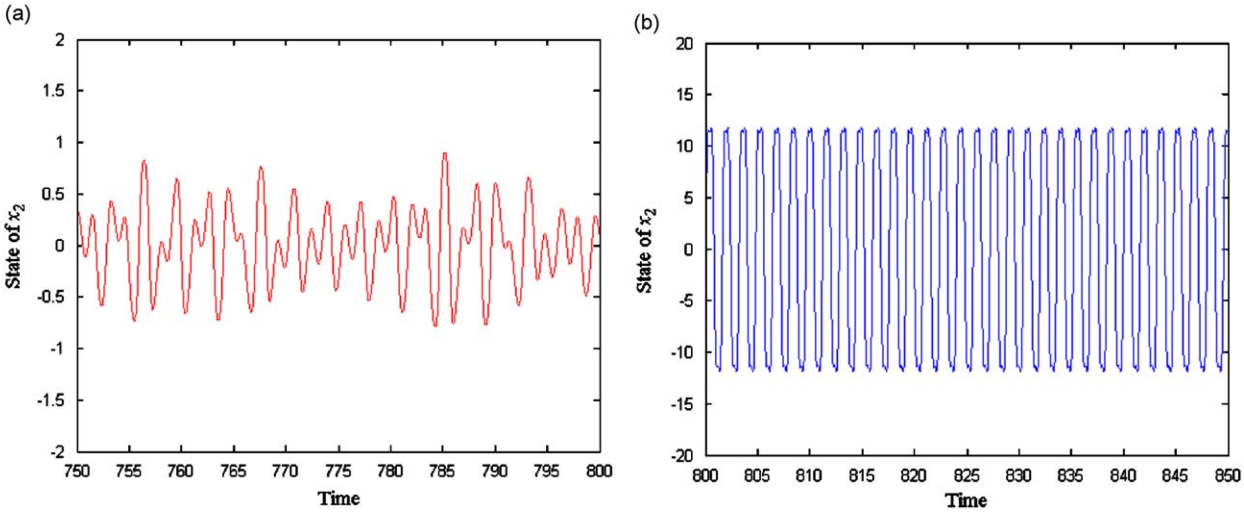

Fig. 3. The velocity responses of the isolated equipment: (a) chaotic under control and (b) periodic without control.

Fig. 5 illustrates the power spectrum of velocity of the isolated equipment in the controlled case as well as the uncontrolled case. It can be seen that both the intensity of line spectra at the primary harmonic frequencies and the whole spectrum of the isolated equipment in the controlled case are much less than those of the periodic motion in the uncontrolled case. The notable reduction of the whole power spectrum implies that the capability of vibration isolation under control has been improved considerably as well.

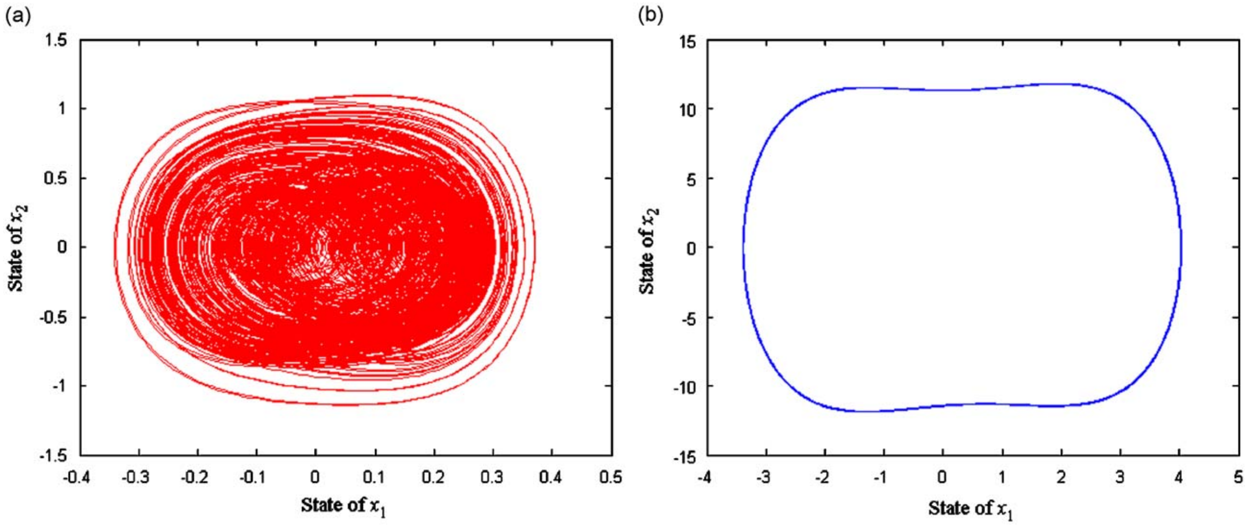

Fig. 4. The phase portrait of the isolated equipment: (a) with control and (b) without control.

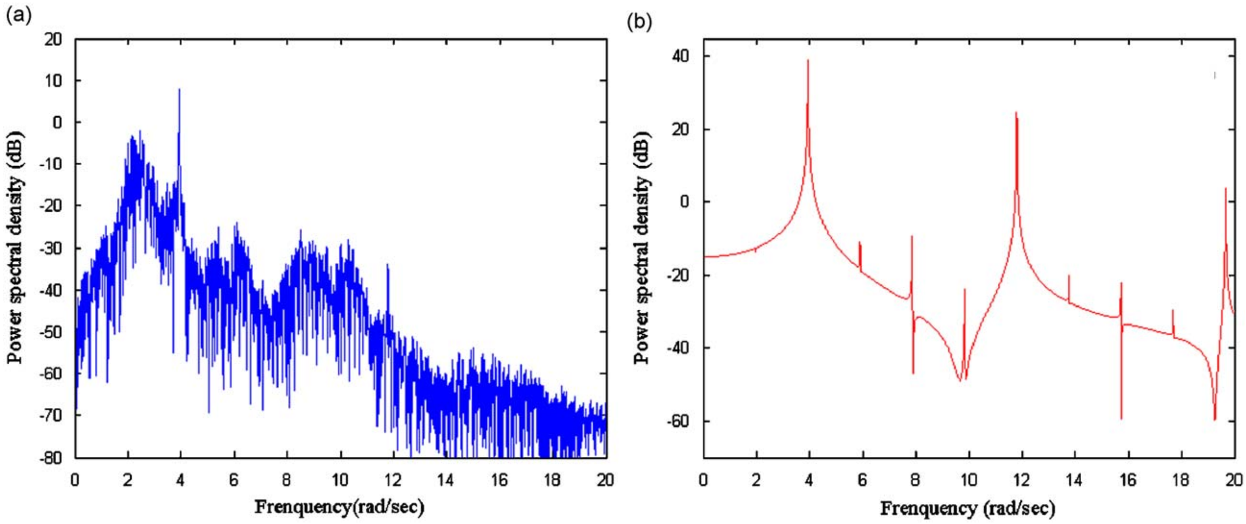

Fig. 5. The power spectrum of velocity of the isolated equipment: (a) with control and (b) without control.

As mentioned above, the gains λ_1 and λ_2 play a role in speeding up the speed of synchronization and the scaling factors α_1 and α_2 are used to proportionally diminish the vibration amplitudes. We next numerically show the effects of controller parameters on the dynamic performance. The relation of gain λ_1 and synchronization error e_1 is given in Fig. 6. It can be seen that the synchronization error e_1 reaches zero faster as the parameter λ_1 increases, but there is a saturation phenomenon in the change trend when the parameter λ_1 is bigger than 30. Fig. 7 shows the relationship between scaling factor α_1 and the rms of state x_2. The rms of x_2 increases with the increasing of scaling factor α_1, and the correlation is direct proportion nearly. The amplitude's rms of synchronization signal identically matches with the ones of driven signal when $\alpha_1 = 1$. Thus, we can adjust the scaling factors to achieve the desired chaotic motion of vibration isolation system via MPS.

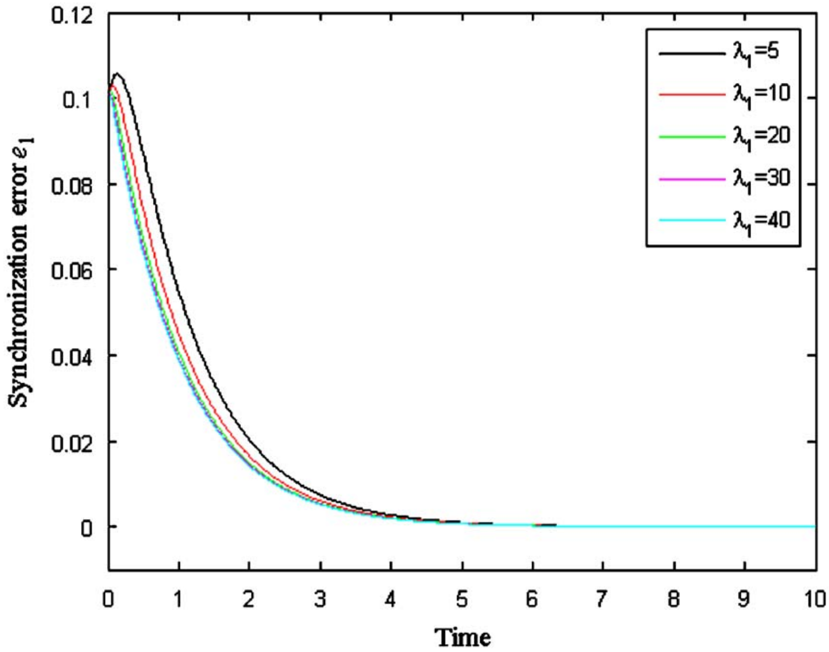

Fig. 6. The relationship of synchronization error e_1 and controller parameters λ_1.

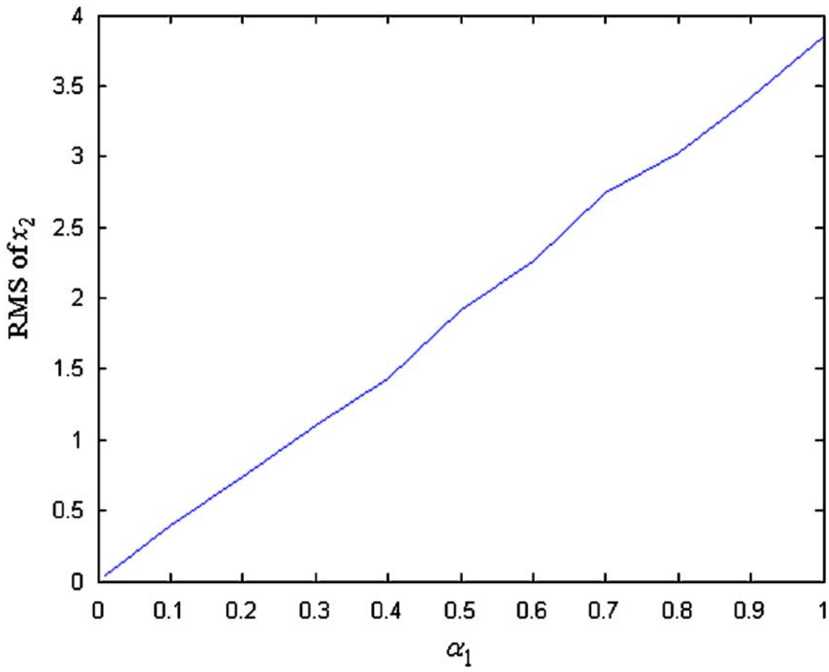

Fig. 7. The relationship of the rms of x_2 with controller parameters α_1.

4. Conclusion

The methodology of vibration control based on MPE to achieve both the reduction of line spectra and the improvement of vibration isolation has been presented in this paper. The chaotifying vibration signals with broad band frequency are utilized to reduce line spectra and the scaling factors of modified projective chaos synchronization enable us to proportionally diminish the vibration amplitudes. The application to the acoustic stealth of submarines has been discussed numerically. The method may be developed into a newly reduction technique of noise and vibration different from the conventional mechanical field as well as civil structure field. The testing experiment to verify the feasibility in practical engineering is our further work in future.

Acknowledgments

The authors would like to express their sincere appreciation to the reviewers for their constructive comments. This work was supported by the National Natural Science Foundation of China (Grant No. 10672054).

References

[1] M. Lakshmanan, K. Murali, *Chaos in Nonlinear Oscillators: Controlling and Synchronization*, World Scientific, Singapore, 1996.
[2] G. Chen, X. Dong, *From Chaos to Order-perspectives, Methodologies, and Applications*, World Scientific, Singapore, 1998.
[3] M. Itoh, H. Murakami, New communication systems via chaotic synchronizations and modulations, *IEICE Transactions on Fundamentals of Electronics* 78-A (1995) 285–290.
[4] C.C. Wang, J.P. Su, A new adaptive structure control for chaotic synchronization and secure communication, *Chaos, Solitons and Fractals* 20 (2004) 967–977.
[5] J. Kawata, Y. Nishio, H. Dedieu, A. Ushida, Performance comparison of communication systems using chaos synchronization, *IEICE Transactions on Fundamentals of Electronics* 82-A (1999) 1322–1328.
[6] M. Wiercigroch, B. de Kraker, *Applied Nonlinear Dynamics and Chaos of Mechanical Systems with Discontinuities*, World Scientific, Singapore, 2000.
[7] J.-J. Lou, S.-J. Zhu, L. He, X. Yu, Application of chaos method to line spectra reduction, *Journal of Sound and Vibration* 286 (3) (2005) 645–652.
[8] S.-Y. Liu, X. Yu, S.-J. Zhu, Study on the chaos anti-control technology in nonlinear vibration isolation system, *Journal of Sound and Vibration* 310 (4–5) (2008) 855–864.
[9] X. Yu, S.-J. Zhu, S.-Y. Liu, A new method for line spectra reduction similar to generalized synchronization of chaos, *Journal of Sound and Vibration* 306 (2007) 835–848.
[10] J. Guckenheimer, P. Holmes, *Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields*, Springer, New York, 1986.
[11] G.L. Wen, J.-H. Xie, Period-doubling bifurcation and non-typical routes to chaos of a two-degree-of-freedom vibro-impact system, *ASME Transactions on Journal of Applied Mechanics* 68 (4) (2001) 670–674.
[12] G.L. Wen, Complicated Bifurcation Phenomena and Chaotic Behaviors of Vibro-impact Systems, PhD Thesis, Southwest Jiaotong University, 2001 (in Chinese).
[13] L.M. Pecora, T.L. Carrol, Synchronization in chaotic systems, *Physical Review Letters* 64 (8) (1990) 821–825.
[14] J.H. Park, Adaptive modified projective synchronization of a unified chaotic system with an uncertain parameter, *Chaos, Solitons and Fractals* 34 (2007) 1552–1559.
[15] G.H. Li, Modified projective synchronization of chaotic system, *Chaos, Solitons and Fractals* 32 (5) (2007) 1786–1790.
[16] R. Mainieri, J. Rehacek, Projective synchronization in three-dimensional chaotic systems, *Physical Review Letters* 82 (15) (1999) 3042–3045.
[17] D. Xu, Control of projective synchronization in chaotic systems, *Physical Review E* 63 (2) (2001) 027201.
[18] G.L. Wen, D. Xu, Nonlinear observer control for full-state projective synchronization in chaotic continuous-time systems, *Chaos, Solitons and Fractals* 26 (2005) 71–77.
[19] J.H. Park, Chaos synchronization of between two different chaotic dynamical systems, *Chaos, Solitons and Fractals* 27 (2006) 549–554.
[20] G.L. Wen, Q.-G. Wang, C. Lin, X. Han, G.Y. Li, Synthesis for robust synchronization of chaotic systems under output feedback control with multiple random delays, *Chaos, Solitons and Fractals* 29 (5) (2006) 1142–1146.
[21] G.L. Wen, Q.-G. Wang, C. Lin, G.Y. Li, X. Han, Chaos synchronization via multivariable PID control, *International Journal of Bifurcation and Chaos* 17 (5) (2007) 1753–1758.
[22] M.F. Hu, Z.Y. Xu, Adaptive feedback controller for projective synchronization, *Nonlinear Analysis: Real World Applications* 9 (3) (2008) 1253–1260.
[23] D.J. Wagg, Partial synchronization of non-identical chaotic systems via adaptive control, with applications to modeling coupled nonlinear systems, *International Journal of Bifurcation and Chaos* 12 (3) (2002) 561–570.
[24] H.Y. Du, Q.H. Zeng, C.H. Wang, Function projective synchronization of different chaotic systems with uncertain parameters, *Physics Letters A* 372 (2008) 5402–5410.
[25] H.K. Khalil, *Nonlinear Systems*, Macmillan Publishing Company, New York, 1993.